

**MATHEMATICAL MODELLING FOR THE  
ESTIMATION OF CROP INSURANCE PREMIUM:  
A CASE STUDY OF TOTAL PREMIUM EQUAL  
DAMAGE COMPENSATION**

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**Abstract**

This research aims to study and construct a mathematical model used to estimate the crop insurance premium for the case of total premium equal damage compensation. We would also like to study Thai government's relief giving to agriculturists, who experienced flood. The scope of this research is a crop insurance in Thailand covering crop damage from flood. The result shows that the expected insurance cost (equal premium) for each agriculturist's household could be calculated by the product of an expected compensation value for each flood and an expected number of compensation claim:  $E(W_h) = E(W_c)U(x_0, y_0)$ .

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## 1. Introduction

Crop insurance is a type of insurance made for agriculturists, whose crop are damaged by some natural disasters such as storm, hail, and flood. In Thailand, there was a crop insurance testing during year 1978-1980. However, it was unsuccessful due to several factors such as too few numbers of insured agriculturists, too small insured areas, lack of agriculturists' understanding of crop insurance, and too little amount of premiums to cover all compensations after disasters. Nowadays, Thai government helps agriculturists, who suffer from flood causing their crop damage by paying them an average rate. Thus, there is a damage estimation before government's compensation and a crop insurance for all agriculturists. In other countries, there have been crop insurances for a while. For example, USA starts to have crop insurance in 1938 by founding Federal Crop Insurance Corporation (FCIC). US government owned all of FCIC's stocks. US crop insurance covered the crop damage from disasters and it was voluntary. Thus, only a few US agriculturists paid for crop insurance. After that, there was a legislation to force every agriculturist under central US government in having basic crop insurance by paying only a little fee. Moreover, FCIC offered the optional several crop insurance plans, which central US government and agriculturists shared the premium cost. In Canada, there was a law about crop insurance for natural disasters since 1939. The central government covered most of the premium fee. Later, there was a relieve fund to help agriculturists in case of high damage and a campaign for special agriculturists' saving accounts so that they part of their income were saved for emergency. Loajirachunkul and Chaisilaparungreung [2] had a research on appropriate crop insurances, costs, and plans in Thailand. They suggested that crop insurance in Thailand should begin with the one that covers a capital production. The crop insurance plans could be categorized by types of crops, types of disasters, capital production costs, and agricultural areas. The damage of the crop during cultivated season, in the beginning, includes the crop capital production from natural

disasters. The rates of premiums are determined by agricultural data in the cultivated season from 1988/89 to 1997/98. This information may help the government to provide sufficient budget in order to prevent and resolve the flood problem. The victims of the disasters are adequately compensated. There are many countries have already been using the above strategy such as United State of America, Canada, India, and Australia. Thus, the mathematical model for premium calculation should be studied in order to construct a mathematical model that can be used in the estimation of the damaged crops and the measured relief plan for agriculturists conducted by the government.

## 2. Principle Ideas of the Damaged Crop Models

Miller et al. [3] did a research on an appropriate premium estimation for agriculturists in Georgia. The model is mentioned that the damage and insurance of net premium rate ( $R$ ) equals to a fraction of the damage-estimated value ( $E(L)$ ) and the maximum damage value ( $Y_g$ ):

$$R = E(L) / Y_g. \quad (1)$$

Recently, Babcock and Hart [1] studied actual production history plan and insurance, and premium reduction for agriculturists. For example, the change of income average is equal to the difference between the average compensation value ( $I$ ) and the premium of producer value ( $PP$ ) that is insured for 75% and 65%.

Let  $Y$  be the amount of product of agriculturists and  $p$  be the insured price.

The research indicates that 75% of the premium in actual production history program for corn, soyabean, and wheat is equal to 65% of the premium multiplied by 1.538. The equation is then,

$$\Delta PP = p \times Y \times rate65 \times (1.538 \times 0.75 \times (1 - psub75) - 0.65 \times (1 - psub65)), \quad (2)$$

where  $rate65$  represents 65% premium rate,  $rate75$  represents 75% premium rate,  $psub65$  represents 65% premium subsidy rate, and  $psub75$  represents 75% premium subsidy rate.

The appropriate compensation should be

$$\Delta I = p \times Y \times (0.75 \times rate75 - 0.65 \times rate65). \quad (3)$$

Pongpullponusak et al. [5] did a research on the format and model in insurance mathematics to propose to the Office of Workmen's Compensation Fund in Social Security Office, Ministry of Labour. The research shows that the main issues of consideration of the grant-in-aid and compensated funds are the specific factors that the Office of Workmen's Compensation Fund uses as a determination. The main factors are the chance of risk and value of benefit compensation. The chance of risk is the first factor that determines how much the main grant-in-aid will be collected from employers by the Office of Workmen's Compensation Fund. This depends on the risk of the business company. At the same time, the results of the risk of the beneficiary will determine the value of the beneficial compensation. The amount of money that has to pay for beneficiary is determined from the severity of the danger. The rate of grant-in-aid will be considered by value of beneficial compensation depending on the severity of the damage. The estimation of net grant-in-aid without the cost of management (the grant-in-aid equals compensated money) is as the following.

The estimation of compensated money categorized by types of businesses is calculated from the estimation of grant-in-aid under the assumption that the net grant-in-aid is equal to the estimation of compensated money:

$$\begin{aligned} \text{Net grant-in-aid} &= \frac{\text{Total compensated money}}{\text{Total workers}} \\ &= \frac{\text{Total workers received compensated money}}{\text{Total workers}} \times \frac{\text{Total compensated money}}{\text{Total workers received}} \\ &\qquad\qquad\qquad \text{compensated money} \\ &= \text{Damage rate} \times \text{severity rate}. \end{aligned}$$

Pal and Murthy [4] researched on “An application of Gumbel’s bivariate exponential distribution in estimation of warranty cost of motor cycles.” Under the idea of this insurance, they could estimate the distribution of  $W$  and the renewal function  $U(x, y)$ . The value of estimated insurance  $E(W)$  can be determined by the limited funding insurance policy as follows:

$$W_h = \int_0^{x_0} \int_0^{y_0} W_c(x, y) dU(x, y), \quad (4)$$

where  $W_h$  is the distribution of the renewal procedure,  $W_c(x, y)$  is the compensation value paid to each household, and  $U(x, y)$  is the estimation of number of compensation claims from each household after flood.

Thus, the estimated value of the insurance cost per household of renewal procedure caused by flood is:

$$\begin{aligned} E(W_h) &= E(W_c) \int_0^{x_0} \int_0^{y_0} dU(x, y) \\ &= E(W_c) U(x_0, y_0), \end{aligned}$$

where  $U(x, y) = \sum_{n=1}^{\infty} P(S_n \leq x, S_n^* \leq y)$ , and  $W_c$  does not depend on  $(x, y)$ , but varies to the standard of descriptive statistics calculated from data.

### **3. Mathematical Model Used in Estimating the Premium of the Crop Insurance: A Case Study of Total Premium Equals Damage Compensation**

To construct a mathematical model used for estimating the premium of crop in the case of the total premium equals damage compensation, it will have to consider the limitation and hypothesis, which has to be observed at the main point of the limitation of the information in the past and hypothesis needed to build the model of the problem in crop

insurance via the flood. The main point of the flood is the flood takes a wide area. The problem is that besides the damage of 419,073 households, the report of the damage is only 406,587 households. There are more than 12,486 households (not asking for the compensation from the government because they might not be agriculturists) that lacks of information. Since most of the damaged households in the middle part of the Thailand are caused by the flood after a lot of rain during September through October of every year, the hypothesis can indicate that these households would be flooded and they are beyond the insurance. This is impossible to receive the information from the households, which are not reported. It suggests that the non-agricultural households will not have any problems during the insurance period. Thus, the cost of the insurance will be zero. The consideration will be determined by the statistic records.

For each insurance claim of the households that have flood problem according to the list below:

- Households that ask for insurance claim.
- Expenses of damage estimation.

The model of the estimation for the insurance premium of the flooded crop in case of insurance premium equal damage compensation will be built as the following, given:

- PRE* Insurance premium per household;  
*L* Total damage value;  
*B* Total number of households;  
*N* Total number of damaged households;  
*X* Severity rate of the damage;  
*Y* Damage rate.

$$\text{Therefore, } PRE = \frac{L}{B} = \frac{N}{B} \times \frac{L}{N} = \text{Damage rate} \times \text{Severity rate.}$$

Let  $X = \frac{N}{B}$  be a random variable representing the damage rate during the flood,  $Y = \frac{L}{N}$  be a random variable representing the severity rate of the damage.

If any household asks for the compensation, the government will have to pay a limited amount of compensated money according to the principle of compensation, that is,  $Y \leq y_0$  and  $X \leq x_0$ . This case of principle insurance will provide the compensated money into 2 policies as follow:

- (1) According to the damage insurance policy.
- (2) Limited budget insurance policy.

The objectives of model construction:

- (1) To estimate the insurance premium payment regarded to the real damage of each household.
- (2) To estimate the compensation for the damage regarded to the limited budget.

Limitations of the model:

- (1) The chosen period is the flood period.
- (2) The damage rate during flood will be calculated from the total number of damaged households in the central region of Thailand per total number of households in the central region of Thailand.
- (3) The severity rate of damage will be calculated from the total damage value per total number of households in the central region of Thailand.

For any household, we define  $X$  to be the damage rate in the flood period, and  $Y$  to be the severity rate of the damage. Hence  $X$  and  $Y$  are varied functions. In fact,  $X$  and  $Y$  are the two renewal processes. Consider  $X$  by letting  $T_i$  be the number of damaged households in area  $i$  (assuming each flood damaging  $n$  areas) and letting  $S_n = (T_1 + T_2 + \dots + T_n) / B$ ,

$n \geq 1$ . Thus  $\{S_n, n \geq 1\}$  is a renewal process. Whenever there is a claim for compensated money,  $X$  will be equal to  $S_m$ , where  $m$  is the number of claims occurred at that time. Analogously, let  $T_j^*$  be the damage value at area  $j$  from the beginning to the time when the first compensated money is claimed. Thus,  $S_n^* = (\sum_{i=1}^n T_i^*) / N, n \geq 1$  is another renewal process. As mentioned above, under any insurance policy, the regulation depends on  $X$  and  $Y$ , which are  $x_0$  and  $y_0$ . The claim for the compensated money will be approved, if  $X \leq x_0$  and  $Y \leq y_0$ . Let  $W_c(x, y)$  be the compensation money paid after flood when  $X = x$  and  $Y = y$  (this compensation may depend upon the severity rate of the damage), and let  $U(x, y)$  be a two-variable function of the renewal of  $X$  and  $Y$ . That is  $U(x, y)$  being the expected numbers of claims for compensated money of the insurance until  $X \leq x$  and  $Y \leq y$ . Therefore, the expense for the insurance per one household is  $W_h$  provided by Equation (1). Thus, if the distribution of  $W_c$  and the renewal function  $U(x, y)$  are able to be estimated, the expectation of the expense of the insurance represented by  $E(W_h)$  will be able to be estimated under the limited budget insurance policy.

$$W_h = \int_0^{x_0} \int_0^{y_0} W_c(x, y) dU(x, y), \quad (4)$$

where  $W_h$  is the distribution of the renewal procedure,  $W_c(x, y)$  is the compensation value paid to each household, and  $U(x, y)$  is the estimation of number of compensation claims from each household after flood.

Thus, the estimated value of the insurance cost per household of renewal procedure caused by flood is,

$$E(W_h) = E(W_c) \int_0^{x_0} \int_0^{y_0} dU(x, y)$$

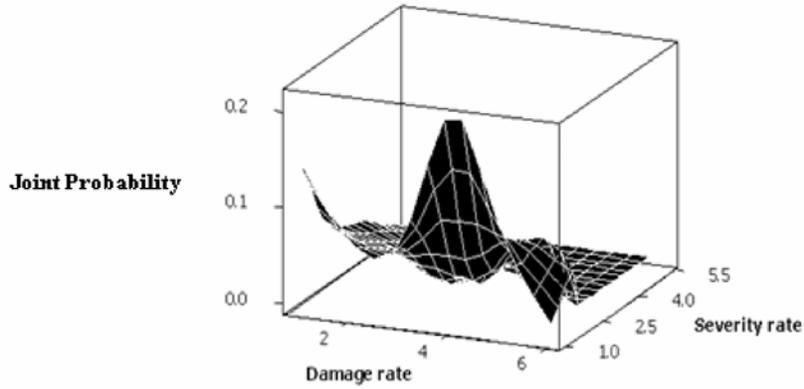
$$= E(W_c)U(x_0, y_0),$$

where  $U(x, y) = \sum_{n=1}^{\infty} P(S_n \leq x, S_n^* \leq y)$ , and  $W_c$  does not depend on  $(x, y)$ , but varies to the standard values of the descriptive statistics.

To find  $P(S_n \leq x, S_n^* \leq y)$ , the bivariate distribution function  $F(t, t^*)$  of  $T_i$  and  $T_j^*$  (for any value of  $i$  and  $j$ , let  $F$  be the cumulative distribution of  $T_i$  and  $T_j^*$ ) has to be known from the study of the damage rate data during the flood and the random variable representing the severity rate of the damage as shown in Table 1 and Figure 1.

**Table 1.** Distribution of frequency and the two ways relative frequency between the damage rate and severity rate of damage during the flood

Damage rate		Severity rate of damage during the flood (baht/household)					Total
		Less than 10,000	10,001- 100,000	100,001- 1,000,000	Less than 10,000	10,001- 100,000	
Less than 0.010	Amount	15	7	5	0	1	28
	Relative frequency	0.13761	0.06422	0.04587	0	0.00917	0.25688
0.011-0.050	Amount	7	5	0	1	0	13
	Relative frequency	0.06422	0.04587	0	0.00917	0	0.11926
0.051-0.100	Amount	9	4	0	0	0	13
	Relative frequency	0.08256	0.03669	0	0	0	0.11926
0.11-0.50	Amount	23	4	2	0	0	29
	Relative frequency	0.21100	0.03669	0.01834	0	0	0.26605
0.51-1.00	Amount	11	8	0	0	0	19
	Relative frequency	0.10091	0.07339	0	0	0	0.17431
More than 1.00	Amount	1	6	0	0	0	7
	Relative frequency	0.00917	0.05504	0	0	0	0.06422
Total	Amount	66	34	7	1	1	109
	Relative frequency	0.60550	0.31192	0.06422	0.00917	0.00917	1



**Figure 1.** It shows the bivariate distribution of the damage rate and the severity rate of damage during the flood.

Figure 1 shows that the damage rate and severity rate have the bivariate exponential distribution. Pal and Murthy [4] specified the types of the distribution as the following:

$$U(x, y) = \sum_{n=1}^{\infty} P(S_n \leq x, S_n^* \leq y),$$

$$U(x, y) = 1 - \exp\left(-\frac{x}{\lambda_x}\right) - \exp\left(-\frac{y}{\lambda_y}\right) + \exp\left[\left(\frac{x}{\lambda_x}\right)^m + \left(\frac{y}{\lambda_y}\right)^m\right]^{\frac{1}{m}}.$$

**3.1. Expense for the estimation of each insurance compensation**

$W_c(x, y)$

The expense of the insurance compensation  $W_c$  consists of the fixed cost and variant cost. There is no information of the insurance expense from the government according to the flood. Thus, the estimation of the expense for the insurance will consist of the following:

**3.1.1. The household expense that has to be estimated from the flood ( $C_{cam}$ )**

$$E(W_c) = E(C_{cam}) = a + bE(N),$$

where  $a$  represents the fixed cost (direct) per household,  $b$  represents the variant cost (indirect) per household, and  $N$  represents the total number of damaged households.

### 3.1.2. The total average number of the damaged households

Let  $N$  be the total number of damaged households,  
 $H$  be the time length of the change from the state  $s(t)$  to  $s(t + k)$ ,  
 $s(t)$  be the number of damaged households at the beginning time  $t$ ,  
 $s(t + k)$  be the number of damaged households during the time  $t + k$ , and  
 $p_0$  be the probability that the damaged households occur between the time  $t$  and  $t + k$ .

Therefore, the cause of the total households' changes will occur between the household  $j$  and the household  $j + 1$ . The process has the average of change from  $s(t)$  to  $s(t + k)$ . When the household  $j$  was damaged, i.e.,  $N = j$ , the cause of change that can be identified will occur during the random time  $T_j$  and  $T_{j+1}$ , so

$$E(N) = \sum_{j=0}^{\infty} jP(N = j) = \sum_{j=0}^{\infty} jP(T_j < T < T_{j+1}).$$

Since the time period of the damage from flood  $T$  has the exponential distribution with parameter  $\lambda$ , the property of the exponential distribution gives us:

$$E(N) = \sum_{j=0}^{\infty} jP(N = j) = \sum_{j=0}^{\infty} jP(T_j < T < T_{j+1}) = \frac{e^{-\lambda h}}{1 - e^{-\lambda h}}.$$

Then by the property of  $H$  as a random variable, which has the probability distribution  $P(H = h_1) = p_0$  and  $P(H = h_2) = 1 - p_0$ , we have

$$E(N) = \frac{e^{-\lambda h_1} p_0 + e^{-\lambda h_2} (1 - p_0)}{1 - e^{-\lambda h_1} p_0 - e^{-\lambda h_2} (1 - p_0)}, \quad (5)$$

where  $h_1$  represents the waiting time until the flood occurs,  $h_2$  represents the waiting time until the ebb,  $p_0$  represents the probability of flooded households, and  $1 - p_0$  represents the probability of non flooded households.

Therefore, from Equation (2), the probability of  $p_0$ , which is the conditional probability, can be calculated as follows:

$$\begin{aligned} p_0 &= P(\text{damage rate during the flood} \mid \text{severity rate of the damage}) \\ &= P(S_n \leq x \mid S_n^* \leq y) = \frac{P(S_n \leq x \cap S_n^* \leq y)}{P(S_n^* \leq y)} = \frac{F(x, y)}{F(y)}, \end{aligned}$$

$$\text{where } F(x, y) = 1 - \exp\left(-\frac{x}{\lambda_x}\right) - \exp\left(-\frac{y}{\lambda_y}\right) + \exp\left[\left(\frac{x}{\lambda_x}\right)^m + \left(\frac{y}{\lambda_y}\right)^m\right]^{\frac{1}{m}}.$$

$$\text{The cumulative function of } y \text{ is } U_y(t) = \sum_{n=1} P(S_n^* \leq t),$$

$$\text{and the marginal function of } y \text{ is } u_y(y) = \frac{dU_y(y)}{dy} = 1 - \left(e^{-\frac{y}{\lambda}}\right).$$

#### 4. Numerical Examples

In order to compute the crop premium rates, we first fix the parameters:  $\lambda = 3$ ,  $a = 0.159081$ ,  $b = 2000$ ,  $h_1 = 0.5$ , and  $h_2 = 0.5$ , while  $X$  and  $Y$  are varied as shown in Table 2. Table 3 shows crop premium rates when  $h_1$  and  $h_2$  are varied, while  $X = 0.5$ ,  $Y = 200,000$ , and other parameters are fixed as above. Table 4 shows the results when  $\lambda$  varies. From the analysis of Table 3, we see that when the damage rate

( $X$ ) increases, the crop premium per household also increases. Similarly, when the severity rate ( $Y$ ) increases, the crop premium rate also increases. From Table 4, if the waiting time period for flood decreases, the insurance premium increases. The waiting time period for ebb does not affect the insurance premium. Lastly from Table 4, if the average waiting time until the damage of flood occurring increases, the insurance premium will decrease.

**Table 2.** Crop premium rates based on damage rates and severity rates

Severity rate ( $Y$ )	Damage rate( $X$ )	Crop premium (baht/household)	Damage rate( $Y$ )	Severity rate( $X$ )	Crop premium (baht/household)
200,000	0.1	175.70	300,000	0.1	175.70
	0.2	297.67		0.2	297.67
	0.3	382.35		0.3	382.35
	0.4	441.13		0.4	441.13
	0.5	480.27		0.5	481.94
	0.6	480.92		0.6	510.27
	0.7	480.92		0.7	529.94
	0.8	480.92		0.8	536.77
	0.9	480.92		0.9	536.77
400,000	0.1	175.70	500,000	0.1	175.70
	0.2	297.67		0.2	297.67
	0.3	382.35		0.3	382.35
	0.4	441.13		0.4	441.13
	0.5	481.94		0.5	481.94
	0.6	510.27		0.6	510.27
	0.7	529.94		0.7	529.94
	0.8	543.59		0.8	543.59
	0.9	553.07		0.9	553.07

**Table 3.** Crop premium rates based on the waiting times until the flood occurs ( $h_1$ ) and the waiting times until the ebb ( $h_2$ )

$h_1$ (days)	$h_2$ (days)	Crop premium rate (baht/household)
1	30	87.60
1	20	87.60
1	10	87.60
0.5	30	479.42
0.5	20	479.42
0.5	10	479.42

**Table 4.** Crop premium rates based on the average waiting times until the damages of the flood occur

Average waiting time until the damage of the flood occurs (days)	Crop premium rate (baht/household)
1	2,577.01
2	973.01
3	480.27
4	261.78
5	149.62
6	87.72
7	52.19

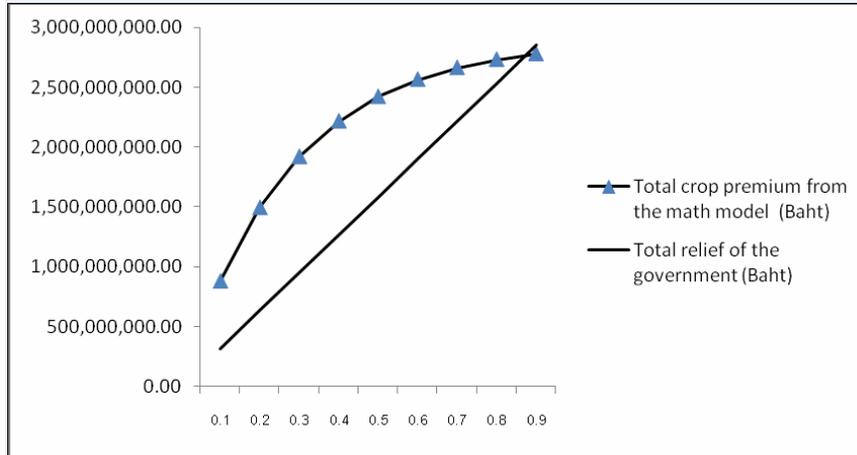
### 5. Conclusion and Suggestion

A comparison between the total crop premium calculated by the mathematical model in the case of total crop premium equal the damage compensation, and the relief of the government are indicated in the Table 5 and Figure 2 below.

**Table 5.** It shows the damage rate, crop premium rate from the mathematical model, the relief of the government, total premium from the mathematical model, and the relief of the government at the severity rate of 500,000 baht/household

Severity rate(Y)	Damage rate(X)	Crop premium rate from the math. model (baht/household)	Relief of the government (baht/household)	Total crop premium from the math. model (baht)	Total relief of the government (baht)
500,000	0.1	175.70	63.09	883,505,165.9	317,247,244.8
	0.2	297.67	126.18	1,496,829,725	634,494,489.7
	0.3	382.35	189.27	1,922,642,004	951,741,734.5
	0.4	441.13	252.36	2,218,216,470	1,268,988,979
	0.5	481.94	315.44	2,423,429,025	1,586,185,939
	0.6	510.27	378.53	2,565,886,061	1,903,433,184
	0.7	529.94	441.62	2,664,796,401	2,220,680,429
	0.8	543.59	504.71	2,733,435,248	2,537,927,674
	0.9	553.07	567.80	2,781,105,305	2,855,174,919

From Figure 2, the mathematical model is curved similar to the second-degree curve. The curve has higher value than the relief of the government line. If the damage rate increases, the model will have a value of insurance premium close to a constant. For the government relief, if the damage rate increases, the relief will be too high for the government to afford. It becomes a huge burden for Thai government to be responsible for the crop insurance after the flood. One way to resolve this problem is that the government should pawn some amount of money equal to the current relief. Then, let Thai agriculturists pay the insurance premium at the difference between the crop premium from the mathematical model and the current relief of the government.



**Figure 2.** It shows total crop premium from the mathematical model (baht) vs. relief of the government (baht) per damage rate ( $x$ -axis).

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